**Workshop on Mathematical Modeling and its Applications in Engineering and Quantum Sciences: A Mathematical Culture in Transition.**
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# Spectral geometry of quantum graphs : an introduction

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**Abstract :** A long-standing topic in the analysis of partial differential equations, and mathematical physics, is the study of how the spectrum (eigenvalues) of a differential operator such as the Laplacian or a Schrödinger operator depends on the geometry or other structural properties of the domain on which they are defined. A classic example is the 19th Century conjecture of Lord Rayleigh, proved by Faber and Krahn in the 1920s, that the ball minimises the lowest eigenvalue of the Laplacian with Dirichlet conditions, among all domains of given volume. Informally, this amounts to saying that "heat loss is slowest in the ball".
In recent years, the study of such eigenvalue optimisation problems has become popular in the specific context of differential operators defined on metric graphs, also known as quantum graphs. Despite being essentially one-dimensional objects, these display surprisingly rich spectral behaviour.
We will start with a gentle introduction to, and apology for, such quantum graphs, and then give examples of how one can use elementary but sometimes tricky arguments to show how the eigenvalues of quantum graph Laplacians depend on metric and geometric properties of the underlying graph.
This talk will be based in part on joint work with multiple colleagues, including Gregory Berkolaiko, Pavel Kurasov, and Delio Mugnolo.

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